

## DARK MATTER AXIONS

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The hypothesis of an ‘invisible’ axion was made by Misha Shifman and others, approximately thirty years ago. It has turned out to be an unusually fruitful idea, crossing boundaries between particle physics, astrophysics and cosmology. An axion with mass of order  $10^{-5}$  eV (with large uncertainties) is one of the leading candidates for the dark matter of the universe. It was found recently that dark matter axions thermalize and form a Bose-Einstein condensate (BEC). Because they form a BEC, axions differ from ordinary cold dark matter (CDM) in the non-linear regime of structure formation and upon entering the horizon. Axion BEC provides a mechanism for the production of net overall rotation in dark matter halos, and for the alignment of cosmic microwave anisotropy multipoles. Because there is evidence for these phenomena, unexplained with ordinary CDM, an argument can be made that the dark matter is axions.

*Keywords:* axion; dark matter; Bose-Einstein condensation

### 1. Introduction

It is a great pleasure and honor to be part of Misha Shifman’s 60th birthday celebration. Among Misha’s many outstanding contributions to particle physics is his well-known proposal, in collaboration with Arkady Vainshtein and Valentine Zakharov, that the axion may be very light and very weakly coupled [1–3]. Here is the abstract of their paper:

*P- and T-invariance violation in quantum chromodynamics due to the so-called  $\theta$ -term is discussed. It is shown that irrespectively of how the confinement works there emerge observable P- and T-odd effects. The proof is based on the assumption that QCD resolves the U(1) problem, i.e., the mass of the singlet pseudoscalar meson does not vanish in the chiral limit. We suggest a modification of the axion scheme which restores the natural P and T invariance of the theory and cannot be ruled out experimentally.*

The  $\theta$ -term mentioned by Shifman et al. is

$$\mathcal{L}_\theta = + \frac{\theta g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \quad (1)$$

where  $G_{\mu\nu}^a$  are the QCD field strengths,  $g$  is the QCD coupling constant and  $\theta$  is a parameter. A  $\theta$ -term is generally present in the action density of the Standard Model of elementary particles [4]. Its existence raises a puzzle, called the Strong CP Problem. As Shifman et al. explain in their paper, the physics of QCD necessarily depends on the value of  $\theta$ , if none of the quark masses vanish, because otherwise QCD wouldn't solve the  $U_A(1)$  problem (of explaining why the mass of the singlet pseudoscalar meson does not vanish in the chiral limit) and hence couldn't be the correct theory of strong interactions. This is an important point. If it were possible for QCD to be independent of  $\theta$ , the Strong CP Problem wouldn't be so urgent.

One can show that QCD physics depends on the value of  $\theta$  only through the combination  $\bar{\theta} \equiv \theta - \arg \det m_q$  where  $m_q$  is the quark mass matrix. If  $\bar{\theta} \neq 0$  the strong interactions violate P and CP. Such P and CP violation is incompatible with the experimental upper bound on the neutron electric dipole moment [5] unless  $|\bar{\theta}| < 10^{-10}$ . In the Standard Model, P and CP violation are introduced by letting the elements of the quark mass matrix  $m_q$  be arbitrary complex numbers [6]. In that case,  $\bar{\theta}$  is of order one. The Strong CP Problem is the problem of explaining why  $|\bar{\theta}| < 10^{-10}$  instead.

The Strong CP Problem is solved if the term (1) in the Standard Model action density is replaced by

$$\mathcal{L}_{\text{axion}} = - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{g^2}{32\pi^2} \frac{\varphi(x)}{f} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \quad (2)$$

where  $\varphi(x)$  is a new scalar field, and  $f$  is a constant with dimension of energy. In the modified theory,  $\bar{\theta} = \frac{\varphi(x)}{f} - \arg \det m_q$  depends on the expectation value of  $\varphi(x)$ . This field settles to a value that minimizes the effective potential. The Strong CP Problem is solved because the minimum of the QCD effective potential  $V(\bar{\theta})$  occurs at  $\bar{\theta} = 0$  [7]. The  $\varphi G \cdot \tilde{G}$  interaction in Eq. (2) is not renormalizable. However, there is a recipe for constructing renormalizable theories whose low energy effective action density is of the form of Eq. (2): construct the theory in such a way that it has a  $U_{PQ}(1)$  symmetry which is a global symmetry of the classical action density, is broken by the color anomaly, and is spontaneously broken. Such a symmetry is called Peccei-Quinn symmetry after its inventors [8]. Weinberg and Wilczek [9] pointed out that a theory with  $U_{PQ}(1)$  symmetry has a light pseudo-scalar particle, called the axion. The axion field is  $\varphi(x)$ .  $f$  is

of order the expectation value that breaks  $U_{PQ}(1)$ , and is called the “axion decay constant”.

The axion mass is given in terms of  $f$  by [9]

$$m \simeq 6 \text{ eV} \frac{10^6 \text{ GeV}}{f}. \quad (3)$$

All axion couplings are inversely proportional to  $f$ . The axion coupling to two photons is:

$$\mathcal{L}_{a\gamma\gamma} = -g_\gamma \frac{\alpha}{\pi} \frac{\varphi(x)}{f} \vec{E} \cdot \vec{B}, \quad (4)$$

where  $\vec{E}$  and  $\vec{B}$  are the electric and magnetic fields,  $\alpha$  is the fine structure constant, and  $g_\gamma$  is a model-dependent coefficient of order one. It had first been thought that  $f$  is of order the electroweak scale, in which case the axion couplings have strength typical of neutrinos and the axion mass is relatively large, in the 10 keV to 10 MeV range. Such axions were quickly ruled out by particle physics (beam dumps and rare decays) and nuclear physics experiments. But Shifman and others showed [1–3] that  $f$  can be made arbitrarily large, and hence the axion can be made arbitrarily light and weakly coupled. Such an axion is unconstrained by the aforementioned experiments, and was dubbed ‘invisible’. May an invisible axion really exist?

The axion has been searched for in many places, and has not been found [10]. Axion masses larger than about 50 keV are ruled out by the aforementioned particle and nuclear physics experiments. The next range of axion masses, in decreasing order, is ruled out by stellar evolution arguments. The longevity of red giants rules out  $200 \text{ keV} > m > 0.5 \text{ eV}$  [11,12] in case the axion has negligible coupling to the electron (such an axion is called ‘hadronic’), and  $200 \text{ keV} > m > 10^{-2} \text{ eV}$  [13] in case the axion has a sizable coupling to electrons. The duration of the neutrino pulse from supernova 1987a rules out  $2 \text{ eV} > m > 3 \cdot 10^{-3} \text{ eV}$  [14]. Finally, there is a lower limit,  $m \gtrsim 10^{-6} \text{ eV}$ , from cosmology which is discussed in the next section. This leaves open an “axion window”:  $3 \cdot 10^{-3} > m \gtrsim 10^{-6} \text{ eV}$ . The lower edge of this window ( $10^{-6} \text{ eV}$ ) is much softer than its upper edge.

## 2. Axion production in the early universe

There are two populations of axions produced in the early universe, which we may call ‘hot’ (or thermal) and cold. Hot axions are produced in thermal processes such as  $q + g \rightarrow q + a$  where  $q$  is a quark and  $g$  a gluon, or  $\pi + \pi \rightarrow \pi + a$  where  $\pi$  is a pion [15–17]. The number density of thermal

axions today (time  $t_0$ ) is

$$n_a^{\text{th}}(t_0) \simeq \frac{7.5}{\text{cm}^3} \left( \frac{106.75}{\mathcal{N}_D} \right)^{\frac{1}{3}} \quad (5)$$

where  $\mathcal{N}_D$  is the effective number of thermal degrees of freedom at the time axions decouple from the thermal bath. The Standard Model has  $\mathcal{N}_D = 106.75$ . Thermal axions are a form of hot dark matter, similar to neutrinos, in the context of large scale structure formation.

The cold axions are produced when the potential  $V(\bar{\theta})$ , and hence the axion mass, turns on near the QCD phase transition [18]. The critical time, defined by  $m(t_1)t_1 = 1$ , is  $t_1 \simeq 2 \cdot 10^{-7} \text{ sec } (f/10^{12}\text{GeV})^{\frac{1}{3}}$ . Cold axions are the quanta of oscillation of the axion field that result from the turn on of the axion mass. The average number density of cold axions at time  $t_1$  is

$$n_a(t_1) \simeq \frac{1}{2}m(t_1)\langle\varphi^2(t_1)\rangle \simeq \pi f^2 \frac{1}{t_1} \quad . \quad (6)$$

In Eq. (6), we used the fact that the axion field  $\varphi(x)$  is approximately homogeneous on the horizon scale  $t_1$ , because wiggles in  $\varphi(x)$  which entered the horizon long before  $t_1$  have been red-shifted away [19]. We also used the fact that the initial departure of  $\varphi(x)$  from the CP conserving minimum is of order  $f$ . The axions of Eq. (6) are non-relativistic. Assuming that the ratio of the axion number density to the entropy density is constant from time  $t_1$  till today, one finds [18]

$$\Omega_a \simeq \frac{1}{2} \left( \frac{f}{10^{12}\text{GeV}} \right)^{\frac{7}{6}} \left( \frac{0.7}{h} \right)^2 \quad (7)$$

for the ratio of the axion energy density to the critical density for closing the universe.  $h$  is the present Hubble rate in units of 100 km/s.Mpc. The requirement that axions do not overclose the universe implies the constraint  $m \gtrsim 6 \cdot 10^{-6} \text{ eV}$ . For a more detailed discussion of the production and properties of dark matter axions, the reader may wish to consult refs. [17, 20].

### 3. Dark matter caustics

It has been established from a variety of observational inputs that approximately 23% of the energy density of the universe is “cold dark matter” (CDM). The CDM particles must be non-baryonic, and cold. “Cold” means that their primordial velocity dispersion is small enough that it can be set equal to zero for all practical purposes when discussing the formation of

large scale structure. The leading candidates for the CDM particles are axions, weakly interacting massive particles (WIMPs), e.g. the neutralino in supersymmetric extensions of the Standard Model, and sterile neutrinos with mass in the keV range.

A central problem in dark matter studies is the question how CDM is distributed in the halos of galaxies, and in particular in the halo of our Milky Way galaxy. Indeed, knowledge of this distribution is essential for understanding galactic dynamics and for predicting signals in direct and indirect searches for dark matter on Earth.

Galactic halos are thought to be collisionless fluids and must therefore be described in 6-dimensional phase space. A full description gives the phase space distribution  $f(\vec{r}, \vec{v}; t)$  of the dark matter particles in the halo, i.e. their velocity ( $\vec{v}$ ) distribution at every position  $\vec{r}$ . An important simplification occurs in the case of *cold* dark matter because CDM particles lie in phase space on a thin 3-dimensional hypersurface. This fact implies that the velocity distribution is everywhere discrete [21] and that there are surfaces in physical space, called caustics, where the density of dark matter is very large.

Galactic halos have two types of caustics, outer and inner. The outer caustics are simple fold ( $A_2$ ) catastrophes located on nested topological spheres surrounding the galaxy. The catastrophe structure of the inner caustics depends on the angular momentum distribution of the infalling dark matter particles. If that angular momentum distribution is dominated by net overall rotation, implying  $\vec{\nabla} \times \vec{v} \neq 0$ , the inner caustics are a set of ‘tricuspid rings’. A tricuspid ring is a closed tube whose cross section is a section of the *elliptic umbilic* ( $D_{-4}$ ) catastrophe [22–24]. The rings are located in the plane of the galaxy. In the self-similar infall model [25], generalized to include the effect of angular momentum [26,27], the caustic ring radii  $a_n$  ( $n = 1, 2, 3, \dots$ ) are predicted to obey the law [22,27]

$$a_n \simeq \frac{40 \text{ kpc}}{n} \left( \frac{v_{\text{rot}}}{220 \text{ km/s}} \right) \left( \frac{j_{\text{max}}}{0.18} \right) \quad , \quad (8)$$

where  $v_{\text{rot}}$  is the rotation velocity of the galaxy and  $j_{\text{max}}$  is a parameter characterizing the amount of angular momentum that the dark matter particles carry.

Observational evidence for caustic rings at the radii  $a_n$  predicted by Eq. (8) was found in the rotation curves of external galaxies [28], the rotation curve of our own galaxy [29], and an IRAS map of the Galactic plane in the direction of the nearest caustic ring ( $n = 5$ ) [29]. A summary of the evidence can be found in ref. [27]. The evidence implies that the distribution of

$j_{\max}$  values over nearby spiral galaxies, including the Milky Way, is peaked at  $j_{\max} \simeq 0.18$ . It also implies that we on Earth are close to a cusp in the nearest caustic ring of dark matter. As a result, the dark matter velocity distribution on Earth is dominated by a single flow, of known velocity vector. That single flow, called the “Big Flow” has density of order 1 GeV/cc, which is two or three times larger than the commonly cited estimates of the *total* local dark matter density.

Finally, the evidence for caustic rings of dark matter halos implies that the dark matter particles fall in with net overall rotation, and hence that their velocity field has non-zero curl:  $\vec{\nabla} \times \vec{v} \neq 0$ . If their velocity field were irrotational ( $\vec{\nabla} \times \vec{v} = 0$ ), the inner caustics would have a tent-like structure [24] which is quite distinct from that of the caustic rings for which evidence was found.

This raises a puzzle. Indeed if the dark matter is cold and collisionless, as is the case for weakly interacting massive particles (WIMPs) and was thought to be the case for axions, the velocity field remains irrotational at all times because it is the outcome of gravitational forces proportional to the gradient of the Newtonian potential [24]. (General relativistic effects allow the creation of rotational velocity fields but are subdominant because the velocities involved are much less than the speed of light.) Thus, if the dark matter is cold and collisionless, one expects the inner caustics of galactic halos to be the tent-like structures of the  $\vec{\nabla} \times \vec{v} = 0$  case, instead of the rings for which evidence was found. This puzzle has bothered me for a number of years, but Qiaoli Yang and I may now have found a solution to it [30]. As explained below, cold dark matter axions form a Bose-Einstein condensate. As a result, their properties differ from those of ordinary CDM.

#### 4. Bose-Einstein condensation of dark matter axions

<sup>a</sup>

The number density of cold axions implied by Eq. (6) is

$$n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{5}{3}} \left( \frac{a(t_1)}{a(t)} \right)^3 \quad (9)$$

where  $a(t)$  is the cosmological scale factor. Because the axion momenta are of order  $\frac{1}{t_1}$  at time  $t_1$  and vary with time as  $a(t)^{-1}$ , the velocity dispersion

<sup>a</sup>All the material in this section is taken from ref. [30].

of cold axions is

$$\delta v(t) \sim \frac{1}{mt_1} \frac{a(t_1)}{a(t)} \quad (10)$$

if each axion remains in whatever state it is in, i.e. if axion interactions are negligible. Let us refer to this case as the limit of decoupled cold axions. If decoupled, the average state occupation number of cold axions is

$$\mathcal{N} \sim n \frac{(2\pi)^3}{\frac{4\pi}{3}(m\delta v)^3} \sim 10^{61} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{8}{3}}. \quad (11)$$

Clearly, the effective temperature of cold axions is much smaller than the critical temperature

$$T_c = \left( \frac{\pi^2 n}{\zeta(3)} \right)^{\frac{1}{3}} \simeq 300 \text{ GeV} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{5}{9}} \frac{a(t_1)}{a(t)} \quad (12)$$

for BEC. Axion number violating processes, such as their decay to two photons, occur only on time scales vastly longer than the age of the universe. The only condition for axion BEC that is not clearly satisfied is thermal equilibrium.

Axions are in thermal equilibrium if their relaxation rate  $\Gamma$  is large compared to the Hubble expansion rate  $H(t) = \frac{1}{2t}$ . At low phase space densities, the relaxation rate is of order the particle interaction rate  $\Gamma_s = n\sigma\delta v$  where  $\sigma$  is the scattering cross-section. Axions have self interactions described by the action density  $\mathcal{L}_{\text{self}} = +\frac{1}{4}\lambda\varphi^4$  where  $\lambda \simeq 0.35(\frac{m}{f})^2$ . The cross-section for  $\varphi + \varphi \rightarrow \varphi + \varphi$  scattering due to axion self interaction is *in vacuum*

$$\sigma_0 = \frac{1}{64\pi} \frac{\lambda^2}{m^2} \simeq 1.5 \cdot 10^{-105} \text{ cm}^2 \left( \frac{m}{10^{-5} \text{ eV}} \right)^6. \quad (13)$$

If one substitutes  $\sigma_0$  for  $\sigma$ ,  $\Gamma_s$  is found much smaller than the Hubble rate, by many orders of magnitude. However, in the cold axion fluid background, the scattering rate is enhanced by the average quantum state occupation number of both final state axions,  $\sigma \sim \sigma_0 \mathcal{N}^2$ , because energy conservation forces the final state axions to be in highly occupied states if the initial axions are in highly occupied states. In that case, the relaxation rate is multiplied by *one* factor of  $\mathcal{N}$  [31]

$$\Gamma \sim n \sigma_0 \delta v \mathcal{N}. \quad (14)$$

Combining Eqs. (9-11,13), one finds  $\Gamma(t_1)/H(t_1) \sim \mathcal{O}(1)$ , suggesting that cold axions thermalize at time  $t_1$  through their self interactions, but only barely so.

It may seem surprising that the huge and tiny factors on the RHS of Eq. (14) cancel each other. In fact the cancellation is not an accident. Consider a generic axion-like particle (ALP) whose mass  $m$  and decay constant  $f$  are unrelated to each other. Its self interaction coupling strength  $\lambda \sim \frac{m^2}{f^2}$ . Cold ALPs appear at a time  $t_1 \sim \frac{1}{m}$  with number density  $n(t_1) \sim f^2 m$ , and velocity dispersion  $\delta v(t_1) \sim 1$ . Substituting these estimates in Eqs. (11), (13) and (14), one finds that the thermalization rate is of order the Hubble rate at  $t_1$ , for all  $f$  and  $m$ .

A critical aspect of axion BEC phenomenology is whether the BEC continues to thermalize after it has formed. Axion BEC means that (almost) all axions go to one state. However, only if the BEC continually rethermalizes does the axion state track the lowest energy state.

The particle kinetic equations that yield Eq. (14) are valid only when the energy dispersion  $\frac{1}{2}m(\delta v)^2$  is larger than the thermalization rate [31]. After  $t_1$  this condition is no longer satisfied. One enters then a regime where the relaxation rate due to self interactions is of order

$$\Gamma_\lambda \sim \lambda n m^{-2} \quad . \quad (15)$$

$\Gamma_\lambda(t)/H(t)$  is of order one at time  $t_1$  but decreases as  $t a(t)^{-3}$  afterwards. Hence, self interactions are insufficient to cause axion BEC to rethermalize after  $t_1$  even if they cause axion BEC at  $t_1$ . However gravitational interactions, which are long range, come in to play. The relaxation rate due to gravitational interactions is of order

$$\Gamma_g \sim G n m^2 \ell^2 \quad (16)$$

where  $\ell \sim (m\delta v)^{-1}$  is the correlation length.  $\Gamma_g(t)/H(t)$  is of order  $4 \cdot 10^{-8} (f/10^{12} \text{ GeV})^{\frac{2}{3}}$  at time  $t_1$  but grows as  $ta^{-1}(t) \propto a(t)$ . Thus gravitational interactions cause the axions to thermalize and form a BEC when the photon temperature is of order 100 eV  $(f/10^{12} \text{ GeV})^{\frac{1}{2}}$ .

The process of axion Bose-Einstein condensation is constrained by causality. Thus one expects overlapping condensate patches with typical size of order the horizon. As time goes on, say from  $t$  to  $2t$ , the axions in  $t$ -size condensate patches rethermalize into  $2t$ -size patches. The correlation length is then of order the horizon at all times, implying  $\delta v \sim \frac{1}{mt}$  instead of Eq. (10), and  $\Gamma_g/H \propto t^3 a^{-3}(t)$  after the BEC has formed. Therefore gravitational interactions rethermalize the axion BEC on ever shorter time scales compared to the age of the universe.

The axion field may be expanded in modes labeled  $\vec{\alpha}$ :

$$\varphi(x) = \sum_{\vec{\alpha}} [a_{\vec{\alpha}} \Phi_{\vec{\alpha}}(x) + a_{\vec{\alpha}}^\dagger \Phi_{\vec{\alpha}}^*(x)] \quad (17)$$



where the  $\Phi_{\vec{\alpha}}(x)$  are the positive frequency c-number solutions of the Heisenberg equation of motion for the axion field

$$D^\mu D_\mu \varphi(x) = g^{\mu\nu} [\partial_\mu \partial_\nu - \Gamma_{\mu\nu}^\lambda \partial_\lambda] \varphi(x) = m^2 \varphi(x) \quad , \quad (18)$$

and the  $a_{\vec{\alpha}}$  and  $a_{\vec{\alpha}}^\dagger$  are creation and annihilation operators satisfying canonical commutation relations. We neglect the self-interaction term which would otherwise appear on the RHS of Eq. (18), because it is of order  $\frac{\rho}{f^2} \varphi$ , where  $\rho$  is the axion density, and hence smaller by the factor  $\left(\frac{a(t_1)}{a(t)}\right)^3 \frac{t}{t_1}$  than the relevant terms (of order  $\frac{m}{t} \varphi$ ) in that equation. BEC means that all cold axions, except for a small fraction, go to a single state which we label  $\vec{\alpha} = 0$ . The corresponding  $\Phi_0(x)$  is the axion wavefunction. In the spatially flat, homogeneous and isotropic Robertson-Walker space-time,

$$\Phi_0 = \frac{A}{a(t)^{\frac{3}{2}}} e^{-imt} \quad (19)$$

where  $A$  is a constant. The state of the axion field is  $|N\rangle = (1/\sqrt{N!}) (a_0^\dagger)^N |0\rangle$  where  $|0\rangle$  is the empty state, defined by  $a_{\vec{\alpha}} |0\rangle = 0$  for all  $\vec{\alpha}$ , and  $N$  is the number of axions.

To compare axion BEC with CDM, let us divide the observations into three arenas: 1) the behaviour of density perturbations on the scale of the horizon, 2) their behaviour during the linear regime of evolution within the horizon, and 3) their behaviour during the non-linear regime. CDM provides a very successful description in arena 2. However, axion BEC and CDM are indistinguishable in arena 2 on all scales of observational interest [30,32]. In particular, the equation governing the evolution of axion BEC perturbations is

$$\partial_t^2 \delta + 2H \partial_t \delta - \left( 4\pi G \rho_0 - \frac{k^4}{4m^2 a^4} \right) \delta = 0 \quad (20)$$

where  $k$  is co-moving wavevector. The last term in Eq. (20) is absent for CDM. Eq. (20) implies that the axion BEC has Jeans length

$$\begin{aligned} k_J^{-1} &= (16\pi G \rho m^2)^{-\frac{1}{4}} \\ &= 1.02 \cdot 10^{14} \text{ cm} \left( \frac{10^{-5} \text{ eV}}{m} \right)^{\frac{1}{2}} \left( \frac{10^{-29} \text{ g/cm}^3}{\rho} \right)^{\frac{1}{4}} . \end{aligned} \quad (21)$$

However, the Jeans length is small compared to the smallest scales ( $\sim 100$  kpc) for which we have observations on the behavior of density perturbations in the linear regime.

In the non-linear regime of structure formation (arena 3) and in the absence of rethermalization, axion BEC and CDM again differ only on

length scales smaller than de Broglie wavelength. This follows from the WKB approximation and has also been shown by numerical simulation [33]. Since the axion de Broglie wavelength (of order 10 meters in galactic halos) is negligibly small compared to all length scales of observational interest, we again find that axion BEC and CDM are indistinguishable when there is no rethermalization of the BEC.

However, it was found above that gravitational interactions do rethermalize the axion BEC continually so that the axion state tracks the lowest energy state. This is relevant to the angular momentum distribution of dark matter axions in galactic halos. The angular momentum of galaxies is caused by the gravitational torque of nearby galaxies early on when protogalaxies are still close to one another [34]. As was mentioned in Section III, CDM presents us with a puzzle. The velocity field of ordinary cold dark matter, such as WIMPs, remains irrotational whereas the evidence for caustic rings of dark matter implies that the dark matter falls in with net overall rotation. The puzzle is solved if the dark matter is an axion BEC which rethermalizes while tidal torque is applied to it. Indeed, the lowest energy state for given total angular momentum is one in which each particle carries an equal amount of angular momentum. In that case there is net overall rotation.  $\vec{\nabla} \times \vec{v} \neq 0$  is accommodated in the BEC through the appearance of vortices. The phenomenon is observed in quantum liquids and well understood [35].

Finally let's consider the behaviour of density perturbations as they enter the horizon (arena 1). Here too axion BEC differs from CDM. The CDM perturbations evolve linearly at all times. The axion BEC perturbations do not evolve linearly when they enter the horizon because the condensates which prevailed in neighboring horizon volumes rearrange themselves, through their gravitational interactions, into a new condensate for the expanded horizon volume. This produces local correlations between modes of different wavevector since the perturbation of wavevector  $\vec{k}$ , upon entering the horizon, is determined by the perturbations of wavevector say  $\frac{1}{2}\vec{k}$  in its neighborhood. We propose this as a mechanism for the alignment of CMBR anisotropy multipoles [36] through the integrated Sachs-Wolfe (ISW) effect. Unlike CDM, the ISW effect is large in axion BEC because the Newtonian potential  $\psi$  changes entirely after entering the horizon in response to the rearrangement of the axion BEC.

## 5. References

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